ML #5: Square and Cube Roots, Estimating Square Roots and Solving Equations with Square Roots

(Math 7 Plus – Unit 4)

In Search of Perfect Squares

1. Use your Geoboard paper to model each square and complete the table below.

SIDE LENGTH	AREA	PERIMETER
5 units		
8 units		
	49 square units	
	4 square units	
		12 units
		24 units

2. Now try to complete the following table, without drawing the squares.

SIDE LENGTH	AREA	PERIMETER
4 units		
14 units		
13 units		
	81 square units	
	100 square units	
		4 units
		44 units
15 units		
12 units		

3. A number is called a **"perfect square"** if it represents the area of a square whose side length is a whole number. For example, 25 is a perfect square, because 25 square units represent the area of a square with a side length of 5 units.

Which column shows perfect squares?

4. List the first 15 perfect squares in order from least to greatest.
(Memorize this list)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

I. Squares and Square Roots

- A **perfect square** has two identical integer factors.
- **For example:** $25 = 5 \bullet 5 = 5^2$ **or** $25 = (-5)(-5) = (-5)^2$
- Since $5^2 = 25$ and $(-5)^2 = 25$, both 5 and −5 (can be written as ±5) are the **square roots** of 25.

Identify the square roots of the following perfect squares

1. The square roots of 16 are ____ and ____ because ( )^2 and ( )^2 = _____

2. The square roots of 81 are ____ and ____ because ( )^2 and ( )^2 = _____

3. The square roots of 4 are ____ and ____ because ( )^2 and ( )^2 = _____

4. The square roots of 169 are ____ and ____ because ( )^2 and ( )^2 = _____

5. The square roots of 36 are ____ and ____ because ( )^2 and ( )^2 = _____

6. The square roots of 9 are ____ and ____ because ( )^2 and ( )^2 = _____

7. The square roots of 1 are ____ and ____ because ( )^2 and ( )^2 = _____

8. The square roots of 225 are ____ and ____ because ( )^2 and ( )^2 = _____

9. The square roots of 144 are ____ and ____ because ( )^2 and ( )^2 = _____

10. The square roots of 49 are ____ and ____ because ( )^2 and ( )^2 = _____

11. The square roots of 100 are ____ and ____ because ( )^2 and ( )^2 = _____

12. The square roots of 196 are ____ and ____ because ( )^2 and ( )^2 = _____

13. The square roots of 25 are ____ and ____ because ( )^2 and ( )^2 = _____

14. The square roots of 64 are ____ and ____ because ( )^2 and ( )^2 = _____

15. The square roots of 121 are ____ and ____ because ( )^2 and ( )^2 = _____

- When you press the $\sqrt{\ }$ key on a calculator, only the positive square root appears. This is called the ***principal square root*** of the number (positive square root)

$\sqrt{16} = 4$ $\quad\quad$ $-\sqrt{16} = -4$

$\sqrt{49} =$ $\quad\quad$ $-\sqrt{49} =$

- Use the principal square root when evaluating an expression. **EX:** $\sqrt{25} + \sqrt{36}$

Simplify each expression

1. $\sqrt{49} + 10$ **2.** $30 - \sqrt{16}$ **3.** $\sqrt{150 - 29}$ **4.** $\frac{\sqrt{225}}{5}$ **5.** $3\sqrt{196}$

Solve the following problems involving square roots

1. What is the length of a square tablecloth that has an area of 3600 square centimeters?

2. A square chessboard has an area of 144 square inches. How long is each side of the board?

3. Your bedroom is a perfect square. If you had to order 225 square feet of carpet to cover the floor, how long is each side of your bedroom?

II. CUBE ROOTS

- A **perfect cube** has three identical integer factors.

For example: $8 = 2 \bullet 2 \bullet 2 = 2^3$ **and** $-8 = -2 \bullet -2 \bullet -2 = -2^3$ or $(-2)^3$

- Therefore, 2 is the **cube root** of 8 and -2 is the **cube root** of -8, or rather $\sqrt[3]{\mathbf{8}} = \mathbf{2}$ ***and*** $\sqrt[3]{\mathbf{-8}} = \mathbf{-2}$

Identify the cube root of the following perfect cubes

1. The cube root of 27 is ____ because ( )^3 = _____
2. The cube root of -27 is ____ because ( )^3 = _____
3. The cube root of 216 is ____ because ( )^3 = _____
4. The cube root of -216 is ____ because ( )^3 = _____
5. The cube root of 1 is ____ because ( )^3 = _____
6. The cube root of -1 is ____ because ( )^3 = _____
7. The cube root of -125 is ____ because ( )^3 = _____
8. The cube root of 125 is ____ because ( )^3 = _____
9. The cube root of -64 is ____ because ( )^3 = _____
10. The cube root of 64 is ____ because ( )^3 = _____

First 6 perfect cubes

1	2	3	4	5	6

Simplify each expression

1. $\sqrt[3]{27} + 15$ **2.** $20 - \sqrt[3]{125}$ **3.** $\sqrt[3]{\frac{1}{64}}$ **4.** $\frac{\sqrt[3]{-216}}{3}$

Solve the following problems involving cube roots

1. What is the side length of a cube that has a volume of 27 cubic centimeters? Show why your answer is correct.

Why would it be unrealistic to ask this same question for a cube with a volume of -27 cubic centimeters?

2. You have a gift box that is a perfect cube. Its volume is 8 cubic inches. How much wrapping paper do you need to cover the box? Give an explanation for your answer.

Would this gift box likely be able to hold Hershey's kisses or a large birthday cake? Justify your answer.

III. ESTIMATING SQUARE ROOTS

List the first 15 perfect squares

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

- You can approximate the value of non-perfect squares using what you know about perfect squares.
- Between which two perfect squares would you find the number 65? _____and _____

  What are their square roots? _____ and _____

- Knowing that, between which two integers would you find $\sqrt{65}$? _____and _____
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Without using your calculator, approximate the value of each of the following square roots by identifying the perfect squares the radicand falls between.

$\sqrt{12}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$ .Therefore, it is between the integers _____ and _____.

$\sqrt{38}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$ .Therefore, it is between the integers _____ and _____.

$\sqrt{130}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$ .Therefore, it is between the integers ____ and _____.

$-\sqrt{29}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$ .Therefore, it is between the integers ____ and _____

$-\sqrt{57}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$ .Therefore, it is between the integers ____ and _____.

IV. SOLVING EQUATIONS WITH SQUARES AND SQUARE ROOTS

- Squares and Square Roots are **INVERSE OPERARTIONS**!!
- **$3^2 = 9$** "square"
- **$\sqrt{9} = 3$** " square root"

- **<u>Solve each of the following equations. Make sure to give the complete answer.</u>**

1. $\sqrt{x} = 10$
2. $\sqrt{x+1} = 4$
3. $\sqrt{x-2} = 3$
4. $\sqrt{81} + 4 = 13$
5. $\sqrt{x} - 6 = 7$
6. $x^2 = 144$
7. $x^2 + 5 = 41$
8. $x^2 - 7 = 93$

- **<u>What do you think about these problems?</u>**

1. $\sqrt[3]{8} = x$
2. $\sqrt[3]{64} + x = \sqrt[3]{343}$
3. $3 = \sqrt[3]{x}$
4. $\sqrt[3]{x} - 1 = 4$
5. $x^3 - 9 = 207$
6. $4 - x^3 = 5$
7. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. If the volume of a given sphere is 2304π, determine the radius of the sphere.